

Color Singlet Strangelets

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When considering strangelets at finite temperature it is important to obey the constraint that any observed state must be color singlet. This constraint results in an increase in strangelet masses as calculated at fixed entropy per baryon. We use the color singlet partition function for an MIT bag, derived using the group theoretical projection method, to calculate strangelet masses. Mean shell effects are included in a liquid drop model, by using a density of states obtained from the multiple reflection expansion. Another important effect of the color singlet restriction, namely many orders of magnitude suppression of thermal nucleation of quark-gluon plasma in collisions, will also briefly be described.

I. INTRODUCTION

The (meta-) stability of strange quark matter as suggested by several authors [1–4] has spurred theoretical as well as experimental activities aimed at answering the question of whether strange quark matter already exists in Nature (*e.g.*, in the cores of neutron stars or in cosmic rays) or can be produced in a laboratory (most likely in relativistic heavy-ion collisions). It is not an easy question to answer with our present theoretical tools, since it involves non-perturbative aspects of the strong interaction. Precisely for this reason most of the theoretical work has been done using different phenomenological models. Ultimately the question of the stability of strange quark matter will have to be answered experimentally/observationally, *e.g.*, by the detection of strangelets in heavy ion collisions. Strangelet production in ultrarelativistic heavy-ion collisions would also constitute an unambiguous signal of quark-gluon plasma formation.

While not making specific predictions for the outcome of an experiment here, we do try to address some of the points which are believed to be important for experimental strangelet searches. These are the effects of small system size and the fact that the produced strangelet is not born cold, but has to survive the hot environment ensuing the collision of two heavy nuclei. In addition to the added thermal energy, which in itself results in an increase of strangelet masses, there is a more subtle effect, *viz.* the fact that the strangelet must be color singlet.

We have used the MIT bag model [5] to describe a strangelet as an ideal Fermi gas of u , d and s quarks (and also antiquarks and gluons at finite temperature). We use a density of states derived using the multiple reflection expansion [6] which gives corrections due to the finite system size. This is the liquid drop model of strangelets, similar to the liquid drop model of nuclei. The overall effects of the shells are well described by the liquid drop model, whereas the detailed effects of the shells, such as shell closures, which are likely to be important at very low baryon number are not revealed in the liquid drop model. However, recent results [7] indicate that shell effects are washed out at elevated temperatures, making shell model calculations and liquid drop calculations effectively equivalent. The main advantage of the liquid drop model over the shell model is that many quantities can be evaluated analytically.

Some of the results presented here are described in Ref. [8], where only massless quarks were treated.

II. STRANGELETS AT FINITE TEMPERATURE

We first consider strangelets at non-zero temperatures without inclusion of the color-singlet constraint. In this case we can obtain all relevant information from the grand canonical partition function Z of a noninteracting gas of quarks, antiquarks, and gluons at temperature $T = \beta^{-1}$ in an MIT bag of volume V

$$\ln Z = \sum_i \ln Z_i - \beta B V. \quad (1)$$

Here Z_i is the partition function for particle species i and B is the bag constant, which is the energy density of the perturbative vacuum inside the bag. Quarks, antiquarks, and gluons contribute terms of the form

$$\ln Z_i = \pm g_i \int_0^\infty dk \rho(k) \ln \{1 \pm \exp[-\beta(\epsilon(k) - \mu_i)]\}, \quad (2)$$

where the upper sign is for fermions, and the lower for bosons. g_i is the statistical weight due to the spin (helicity) and color degrees of freedom, $\epsilon(k)$ is the energy of a particle with momentum k , and μ is the chemical potential. We use units in which $\hbar = k_B = c = 1$. The density of states $\rho(k)$ is a smooth function of k calculated in the framework of the multiple reflection expansion [6]. It has the general form

$$\rho(k) = \frac{Vk^2}{2\pi^2} + f_S \left(\frac{m}{k}\right) Sk + f_C \left(\frac{m}{k}\right) C + \dots, \quad (3)$$

where S is the surface area of the bag, and $C \equiv \int_S dS(1/R_1 + 1/R_2)$ is the extrinsic curvature of the bag surface (R_1 and R_2 are the principal radii of curvature). The functions f_S and f_C depend on the equations of motion and on the boundary conditions, but *not* on the geometry of the confining surface, which is only assumed to be smooth. The surface coefficient for a quark field in a cavity with confining MIT bag boundary conditions is [9,10]

$$f_S \left(\frac{m}{k}\right) = -\frac{1}{8\pi} \left(1 - \frac{2}{\pi} \tan^{-1} \frac{k}{m}\right). \quad (4)$$

Note that this expression vanishes in the limit $m \rightarrow 0$, so that massless quarks do not contribute to the surface tension. So for massless (u and d) quarks the lowest order correction to the density of states is the curvature term. The curvature coefficient f_C has not been calculated using the multiple reflection expansion in the general case of massive quarks, but for massless quarks with MIT bag boundary conditions the result is [10] $f_C = -1/24\pi^2$. In the limit of infinite mass the quark can be considered non-relativistic and the field equation reduces to the wave equation. In this case the boundary condition of the MIT bag is the well known Dirichlet boundary condition studied by Balian and Bloch [6] who derive the result $f_C = 1/12\pi^2$. It was shown by Madsen [11] that the expression

$$f_C \left(\frac{m}{k}\right) = \frac{1}{12\pi^2} \left[1 - \frac{3k}{2m} \left(\frac{\pi}{2} - \tan^{-1} \frac{k}{m}\right)\right], \quad (5)$$

which has the above special cases as limits, is in very good agreement with shell model calculations (see also Fig. 1). We will therefore adopt (5) as the curvature coefficient for quarks. For gluons the relevant expressions were given by Mardor and Svetitsky [10]

$$f_S = 0 \quad f_C = -\frac{1}{6\pi^2}. \quad (6)$$

Just as massless quarks, gluons do not contribute to the surface tension.

It is possible to calculate the thermodynamical potential $\Omega = -T \ln Z$ without resorting to numerical evaluation in the two limits: $T \rightarrow 0$ and $m \rightarrow 0$. Since we are mainly interested in the finite temperature regime we will give the result for \mathcal{N}_q massless quark flavors, including antiquarks and gluons, and assuming a common chemical potential μ_q of all quarks (antiquarks have $\mu_{\bar{q}} = -\mu_q$)

$$\begin{aligned} \Omega(T, \mu_q, V, C) = & - \left[\left(\frac{7\mathcal{N}_q}{60} + \frac{8}{45} \right) \pi^2 T^4 + \frac{\mathcal{N}_q}{2} \left(\mu_q^2 T^2 + \frac{\mu_q^4}{2\pi^2} \right) - B \right] V \\ & + \left[\left(\frac{\mathcal{N}_q}{24} + \frac{4}{9} \right) T^2 + \frac{\mathcal{N}_q}{8\pi^2} \mu_q^2 \right] C. \end{aligned} \quad (7)$$

In the following we consider only spherical systems, where $V = 4\pi R^3/3$ and $C = 8\pi R$ are given in terms of the radius R . When fixing the geometry in this manner there is only one independent “shape variable” which we will take to be the volume. A strangelet is in mechanical equilibrium when $\partial\Omega/\partial V|_{\mu_q, T} = 0$. The chemical potential and the temperature are determined by requiring the baryon number A and the entropy per baryon S/A to be fixed. So to find the equilibrium values of T , μ_q and V we simultaneously solve the equations

$$\left(\frac{\partial\Omega}{\partial V} \right)_{\mu_q, T} = 0, \quad \left(\frac{\partial\Omega}{\partial \mu_q} \right)_{T, V} = 3A, \quad -\frac{1}{A} \left(\frac{\partial\Omega}{\partial T} \right)_{\mu_q, V} = \frac{S}{A} \quad (8)$$

The energy (mass) E of the strangelet can then be calculated by use of the identity

$$E = \Omega + TS + 3\mu_q A. \quad (9)$$

As long as only massless quarks are considered this is equivalent to the expression $E = 4BV$ in equilibrium. This is because massless particles satisfy the relativistic equation of state $E = 3PV$, where P is the pressure. In equilibrium the dynamic pressure of the particles is exactly balanced by the bag constant $B = P$, so that adding the vacuum energy BV gives $E = 3BV + BV = 4BV$. Including a massive strange quark this no longer holds. Plots of the energy per baryon as a function of baryon number obtained by solving Eqs. (8) are shown as full lines in Fig. 1.

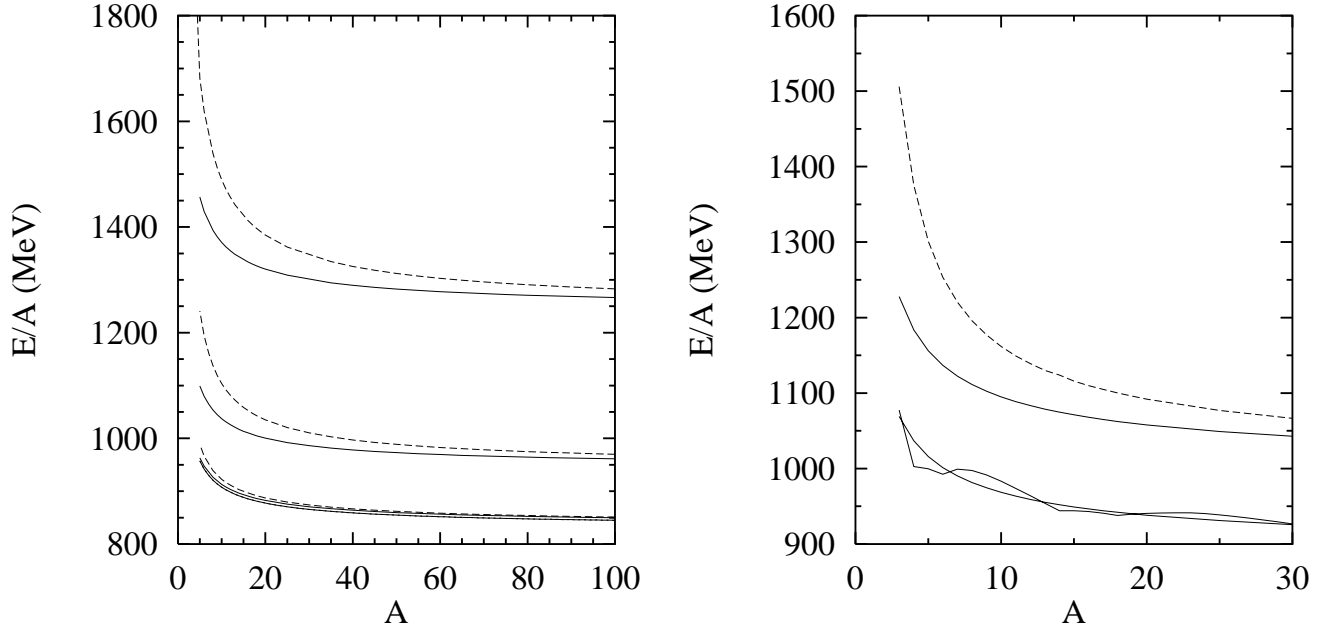


FIG. 1. The energy per baryon number as a function of baryon number. Full curves show results without the color singlet constraint—dashed curves with it. **Left:** curves for fixed entropy per baryon (from top to bottom: $S/A = 10, 5, 1, 0$) with three massless flavors. **Right:** Equivalent results for a strange quark mass of 150 MeV. The two sets of curves correspond to $S/A = 0$ and 5. The jagged curve is a shell model calculation. In both panels $B^{1/4} = 145$ MeV.

III. COLOR SINGLET STRANGELETS

The partition function given by Eqs. (1) and (2) is in fact the grand canonical partition function for a colorless assembly of quarks, antiquarks, and gluons in an MIT bag. In order to force a non singlet color configuration it is necessary to introduce chemical potentials related to the color degree of freedom of the quarks and gluons. Two chemical potentials, corresponding to the two conserved charges (Cartan operators) of $SU(3)$, are needed. Now we need to put in a sum over the different color states of quarks and gluons in the partition function, thus replacing Eq. (2) with

$$\ln Z_i = \pm g'_i \sum_c \int_0^\infty dk \rho(k) \ln \left\{ 1 \pm e^{-\beta[\epsilon(k) - \mu_i - \gamma_1 C_1(c) - \gamma_2 C_2(c)]} \right\}. \quad (10)$$

The statistical weight g'_i no longer includes the color degree of freedom, which is now explicitly summed over. C_1 and C_2 are the eigenvalues of the Cartan operators, which for quarks are in the fundamental representation and for gluons in the adjoint representation. Using the group theoretical projection method developed by Redlich and Turko [12–14], it is possible to construct a canonical partition function (with respect to color) from the grand canonical $Z(\gamma_1, \gamma_2)$ constructed using the prescription just outlined. The resulting color-singlet canonical partition function is

$$Z_{C.S.} = \int_{-\pi}^{\pi} d\theta_1 d\theta_2 M(\theta_1, \theta_2) Z(iT\theta_1, iT\theta_2), \quad (11)$$

where $M(\theta_1, \theta_2) d\theta_1 d\theta_2$ is the Haar measure of $SU(3)$. Note that the chemical potentials $\gamma_i = iT\theta_i$ are now purely imaginary. The baryon number is still treated in a grand canonical ensemble, so $Z_{C.S.}$ is a mixed canonical/grand canonical ensemble.

In the thermodynamic limit ($V \rightarrow \infty$) the two ensembles are equivalent, *i.e.*, they give identical mean values—differing only with respect to fluctuations. For a finite system this is no longer the case, and since color neutrality is exact at all times the canonical prescription is the proper one.

For massless quarks it is possible to evaluate $Z_{\text{C.S.}}$ analytically in a saddle-point approximation, valid at high T and/or μ_q . This was done by Elze and Greiner [15], who for \mathcal{N}_q massless quark flavors obtained the result

$$Z_{\text{C.S.}}(T, V, \mu_q) = \Pi(T, V, \mu_q) Z(T, V, \mu_q), \quad (12)$$

where Π is given by

$$(2\pi\sqrt{3}\Pi)^{-1/4} = VT^3 \left\{ 2 + \mathcal{N}_q \left[\frac{1}{3} + \left(\frac{\mu_q}{\pi T} \right)^2 \right] \right\} + CT \frac{12 - \mathcal{N}_q}{12\pi^2}. \quad (13)$$

Note that Π vanishes in the limit $V \rightarrow \infty$. The limit $T \rightarrow 0$ cannot be taken in the above expression for Π , since the saddle-point approximation breaks down for low T . But for $T \rightarrow 0$ the effect of exact color singletness also disappears since at $T = 0$ it is always possible to construct a color singlet state for $3A$ quarks occupying the lowest energy levels available. For massive quarks we evaluate the partition function numerically.

The effect of the color singlet constraint on the energy per baryon is seen in Fig. 1, where dashed lines include the color singlet constraint. There is a sizeable effect regardless of whether one includes a massive strange quark or not. The main effect of the strange quark mass is to shift curves towards higher E/A . Curves for zero strange quark mass are calculated using the saddle-point approximation, which has been shown to be a good approximation in this case (see Ref. [8]).

IV. COLOR SINGLET SUPPRESSED NUCLEATION

As another example of the effect of color singletness we consider the nucleation of quark matter droplets from hadronic matter using standard homogenous nucleation theory. The low baryon number density region of the phase diagram is where the transition to a quark-gluon plasma would happen in RHIC and LHC experiments. In the region of the phase diagram characterized by high baryon number density and moderate temperature this could be relevant for the nucleation of quark matter droplets in AGS experiments and in the core of neutron stars. In the case of neutron stars the time scales are probably sufficiently large for homogenous nucleation theory to be applicable, whereas this is questionable in the case of quark-gluon plasma formation in heavy ion collisions. An estimate for the nucleation rate is

$$\mathcal{R} \approx T^4 \exp(-\Delta F/T), \quad (14)$$

where ΔF is the height of the free energy barrier that has to be surmounted by the nucleated bubble. For the prefactor we have chosen the dimensional estimate T^4 . One could consider improvements to this estimate but since the relative suppression due to color singlet effects are dominated by the exponential, this is unlikely to change the results significantly.

The free energy barrier is given by the pressure difference between the hadronic phase and the plasma phase (times volume) plus additional contributions from surface tension and the curvature term.

$$\Delta F = (P_h - P_q)V + \sigma S + \gamma C. \quad (15)$$

Note that any contribution from the hadronic phase, *i.e.*, either a non-zero pressure or a surface tension, will increase the barrier height. We will therefore ignore the hadron gas contribution, since it will only augment the effect due to color singletness (for a more elaborate discussion of this point, see Ref. [16]). With this simplification the free energy barrier is simply $\Delta F = \Omega$, where Ω is given by Eq. (7) with $\mathcal{N}_q = 2$. The color singlet constraint is taken into account by using $\Omega = -T \ln Z_{\text{C.S.}}$ as given by Eqs. (12) and (13). The barrier is shown for a particular choice of parameters in Fig. 2.

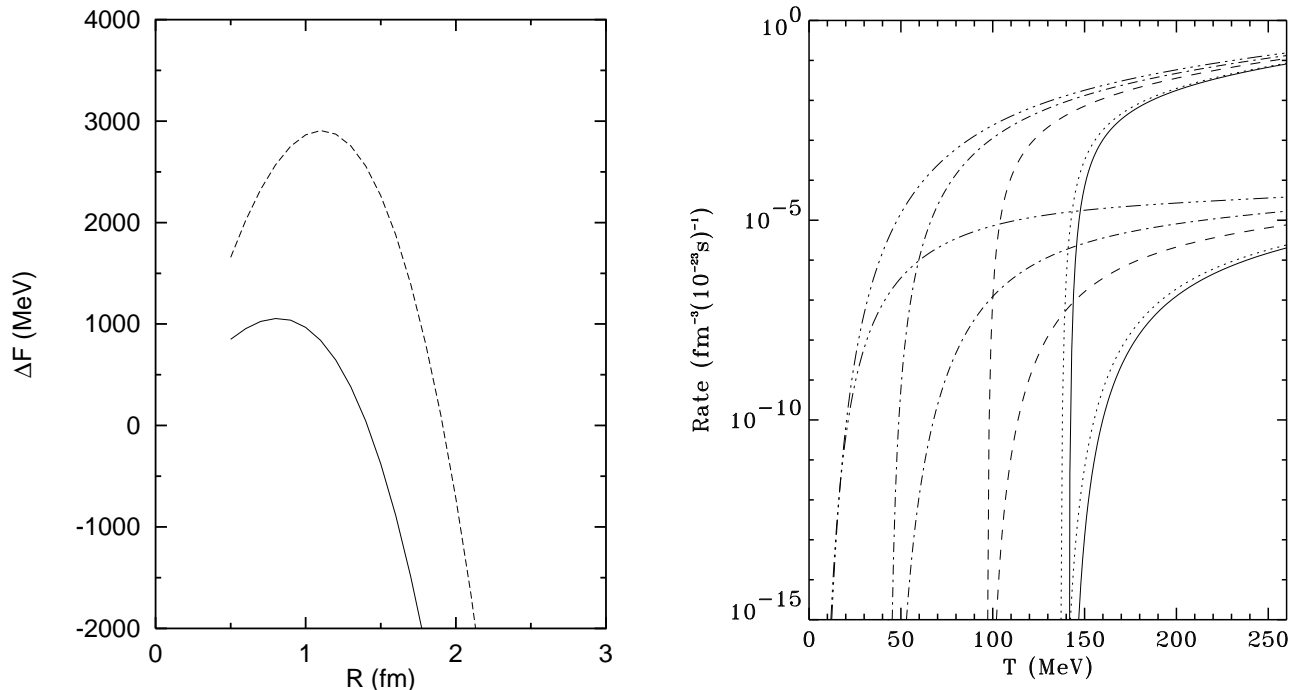


FIG. 2. **Left:** Free energy barrier for $T = 170$ MeV and $\mu_q = 0$. The upper curve is with the singlet constraint, the lower is not. **Right:** Nucleation rate for $\mu_q = 0$ (solid), 100 (dotted), 300 (dashed), 400 (dot-dashed), and 500 MeV (dash-triple dot). Lower curves with singlet constraint, upper without. In both panels $B^{1/4} = 200$ MeV.

Inserting the value of ΔF for a critical bubble in the expression (14) for a range in temperature and chemical potential gives a plot like the one shown in the right panel of Fig. 2, where an extra constraint, namely requiring the droplet to have zero momentum [16], is also included. The effect of the color singlet constraint, which is the dominant of the two constraints, is seen to be quite dramatic. It causes a suppression of the plasma formation rate by four to five orders of magnitude.

V. CONCLUSIONS

We have shown that the effect of exact color singletness plays an important role in strangelets—increasing masses as calculated at fixed entropy per baryon—as well as in the phase transition between hadronic and quark matter, where it causes a suppression of quark gluon plasma nucleation.

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